Part 2 Chapter 6: Trigonometry_complete textbook solutions

📌 Key Formulas & Core Concepts

1. Basic Trigonometric Ratios

For a right-angled triangle with angle θ:

- $\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$
- $\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$
- $\tan\theta = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$
- $\csc\theta = \frac{\text{Hypotenuse}}{\text{Opposite Side}}$
- $\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent Side}}$
- $\cot\theta = \frac{\text{Adjacent Side}}{\text{Opposite Side}}$

2. Reciprocal Relations

- $\csc\theta = \frac{1}{\sin\theta} \Longrightarrow \sin\theta \cdot \csc\theta = 1$
- $\sec\theta = \frac{1}{\cos\theta} \Longrightarrow \cos\theta \cdot \sec\theta = 1$
- $\cot\theta = \frac{1}{\tan\theta} \Longrightarrow \tan\theta \cdot \cot\theta = 1$
- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\cot\theta = \frac{\cos\theta}{\sin\theta}$

3. Fundamental Trigonometric Identities

1. $\sin^2\theta + \cos^2\theta = 1$
   - $\sin^2\theta = 1 - \cos^2\theta$
   - $\cos^2\theta = 1 - \sin^2\theta$
2. $1 + \tan^2\theta = \sec^2\theta$
   - $\sec^2\theta - \tan^2\theta = 1$
   - $\tan^2\theta = \sec^2\theta - 1$
3. $1 + \cot^2\theta = \csc^2\theta$
   - $\csc^2\theta - \cot^2\theta = 1$
   - $\cot^2\theta = \csc^2\theta - 1$

4. Standard Values Table

Ratio / Angle (θ)	0∘	30∘	45∘	60∘	90∘
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\csc\theta$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

5. Applications of Trigonometry (Heights and Distances)

- **Line of Sight:** The line drawn from the eye of an observer to the point in the object viewed by the observer.
- **Angle of Elevation:** The angle formed by the line of sight with the horizontal line when the point being viewed is **above** the horizontal level.
- **Angle of Depression:** The angle formed by the line of sight with the horizontal line when the point being viewed is **below** the horizontal level.

Practice Set 6.1 (Page 131)

1. If $\sin\theta = \frac{7}{25}$, find the values of $\cos\theta$ and $\tan\theta$.

Solution:

Given: $\sin\theta = \frac{7}{25}$

We know that:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{7}{25}\right)^2 + \cos^2\theta = 1$$

$$\frac{49}{625} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{49}{625}$$

$$\cos^2\theta = \frac{625 - 49}{625}$$

$$\cos^2\theta = \frac{576}{625}$$

Taking square root on both sides:

$$\cos\theta = \frac{24}{25}$$

Now,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{7/25}{24/25} = \frac{7}{25} \times \frac{25}{24} = \frac{7}{24}$$

Answer: $\cos\theta = \frac{24}{25}$ and $\tan\theta = \frac{7}{24}$.

2. If $\tan\theta = \frac{3}{4}$, find the values of $\sec\theta$ and $\cos\theta$.

Solution:

Given: $\tan\theta = \frac{3}{4}$

We know that:

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \left(\frac{3}{4}\right)^2 = \sec^2\theta$$

$$1 + \frac{9}{16} = \sec^2\theta$$

$$\sec^2\theta = \frac{16 + 9}{16} = \frac{25}{16}$$

Taking square root on both sides:

$$\sec\theta = \frac{5}{4}$$

We have:

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{5/4} = \frac{4}{5}$$

Answer: $\sec\theta = \frac{5}{4}$ and $\cos\theta = \frac{4}{5}$.

3. If $\cot\theta = \frac{40}{9}$, find the values of $\csc\theta$ and $\sin\theta$.

Solution:

Given: $\cot\theta = \frac{40}{9}$

We know that:

$$1 + \cot^2\theta = \csc^2\theta$$

$$1 + \left(\frac{40}{9}\right)^2 = \csc^2\theta$$

$$1 + \frac{1600}{81} = \csc^2\theta$$

$$\csc^2\theta = \frac{81 + 1600}{81} = \frac{1681}{81}$$

Taking square root on both sides:

$$\csc\theta = \frac{41}{9}$$

We have:

$$\sin\theta = \frac{1}{\csc\theta} = \frac{1}{41/9} = \frac{9}{41}$$

Answer: $\csc\theta = \frac{41}{9}$ and $\sin\theta = \frac{9}{41}$.

4. If $5\sec\theta - 12\csc\theta = 0$, find the values of $\sec\theta$, $\cos\theta$, and $\sin\theta$.

Solution:

Given: $5\sec\theta - 12\csc\theta = 0$

$$5\sec\theta = 12\csc\theta$$

$$\frac{5}{\cos\theta} = \frac{12}{\sin\theta} \quad \left[\because \sec\theta = \frac{1}{\cos\theta}, \csc\theta = \frac{1}{\sin\theta}\right]$$

$$\frac{\sin\theta}{\cos\theta} = \frac{12}{5} \Longrightarrow \tan\theta = \frac{12}{5}$$

We know that:

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \left(\frac{12}{5}\right)^2 = \sec^2\theta$$

$$1 + \frac{144}{25} = \sec^2\theta \Longrightarrow \sec^2\theta = \frac{25 + 144}{25} = \frac{169}{25}$$

Taking square root on both sides:

$$\sec\theta = \frac{13}{5}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{5}{13}$$

We know that:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(\frac{5}{13}\right)^2 = 1 \Longrightarrow \sin^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$$

Taking square root on both sides:

$$\sin\theta = \frac{12}{13}$$

Answer: $\sec\theta = \frac{13}{5}$, $\cos\theta = \frac{5}{13}$, and $\sin\theta = \frac{12}{13}$.

5. If $\tan\theta = 1$, then find the value of $\frac{\sin\theta+\cos\theta}{\sec\theta+\csc\theta}$.

Solution:

Given: $\tan\theta = 1$

Since $\tan 45^\circ = 1$, we have $\theta = 45^\circ$.

Therefore:

- $\sin 45^\circ = \frac{1}{\sqrt{2}}$
- $\cos 45^\circ = \frac{1}{\sqrt{2}}$
- $\sec 45^\circ = \sqrt{2}$
- $\csc 45^\circ = \sqrt{2}$

Substituting these values into the given expression:

$$\frac{\sin 45^\circ + \cos 45^\circ}{\sec 45^\circ + \csc 45^\circ} = \frac{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{\frac{2}{\sqrt{2}}}{2\sqrt{2}} = \frac{2}{\sqrt{2}} \times \frac{1}{2\sqrt{2}} = \frac{1}{2}$$

Answer: $\frac{\sin\theta+\cos\theta}{\sec\theta+\csc\theta} = \mathbf{\frac{1}{2}}$.

6. Prove the following:

(1) $\boldsymbol{\frac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta}$

Proof:

$$\text{LHS} = \frac{\sin^2\theta}{\cos\theta} + \cos\theta$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \sec\theta = \text{RHS}$$

(Hence proved)

(2) $\boldsymbol{\cos^2\theta\left(1 + \tan^2\theta\right) = 1}$

Proof:

$$\text{LHS} = \cos^2\theta\,(1 + \tan^2\theta)$$

$$= \cos^2\theta \cdot \sec^2\theta \quad [\text{Since } 1 + \tan^2\theta = \sec^2\theta]$$

$$= \cos^2\theta \cdot \frac{1}{\cos^2\theta}$$

$$= 1 = \text{RHS}$$

(Hence proved)

(3) $\boldsymbol{\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta}$

Proof:

$$\text{LHS} = \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

Multiplying numerator and denominator inside the square root by $(1 - \sin\theta)$:

$$= \sqrt{\frac{(1 - \sin\theta)(1 - \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}}$$

$$= \sqrt{\frac{(1 - \sin\theta)^2}{1 - \sin^2\theta}}$$

$$= \sqrt{\frac{(1 - \sin\theta)^2}{\cos^2\theta}} \quad [\because 1 - \sin^2\theta = \cos^2\theta]$$

Taking square root:

$$= \frac{1 - \sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \sec\theta - \tan\theta = \text{RHS}$$

(Hence proved)

(4) $(\sec\boldsymbol{\theta} - \cos\boldsymbol{\theta})(\cot\boldsymbol{\theta} + \tan\boldsymbol{\theta}) = \tan\boldsymbol{\theta}\sec\boldsymbol{\theta}$

Proof:

$$\text{LHS} = (\sec\theta - \cos\theta)(\cot\theta + \tan\theta)$$

$$= \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \left(\frac{1 - \cos^2\theta}{\cos\theta}\right)\left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \left(\frac{\sin^2\theta}{\cos\theta}\right)\left(\frac{1}{\sin\theta\cos\theta}\right) \quad [\because 1 - \cos^2\theta = \sin^2\theta \text{ and } \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{\sin\theta}{\cos^2\theta} = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$$

$$= \tan\theta\sec\theta = \text{RHS}$$

(Hence proved)

(5) $\cot\boldsymbol{\theta} + \tan\boldsymbol{\theta} = \csc\boldsymbol{\theta}\sec\boldsymbol{\theta}$

Proof:

$$\text{LHS} = \cot\theta + \tan\theta$$

$$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{1}{\sin\theta\cos\theta} \quad [\because \cos^2\theta + \sin^2\theta = 1]$$

$$= \frac{1}{\sin\theta} \cdot \frac{1}{\cos\theta} = \csc\theta\sec\theta = \text{RHS}$$

(Hence proved)

(6) $\boldsymbol{\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta}$

Proof:

$$\text{LHS} = \frac{1}{\sec\theta - \tan\theta}$$

Rationalizing the denominator by multiplying numerator and denominator by $(\sec\theta + \tan\theta)$:

$$= \frac{1}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}$$

$$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$= \frac{\sec\theta + \tan\theta}{1} \quad [\because \sec^2\theta - \tan^2\theta = 1]$$

$$= \sec\theta + \tan\theta = \text{RHS}$$

(Hence proved)

Practice Set 6.2 (Page 137)

1. A person is standing at a distance of 80 m from a church looking at its top. The angle of elevation is 45°. Find the height of the church.

Solution:

Let C represent the position of the person and AB represent the height of the church.

Distance $BC = 80$ m

Angle of elevation $\angle C = 45^\circ$

In right-angled triangle $\triangle ABC$:

$$\tan 45^\circ = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{AB}{BC}$$

$$1 = \frac{AB}{80}$$

$$AB = 80 \text{ m}$$

Answer: The height of the church is 80 m.

2. From the top of a lighthouse, an observer looking at a ship makes an angle of depression of 60°. If the height of the lighthouse is 90 m, find how far the ship is from the lighthouse. ($\sqrt{3} = 1.73$)

Solution:

Let AB represent the lighthouse of height 90 m, and let C be the position of the ship.

Angle of depression $\angle DAC = 60°$.

Since horizontal line $AD \parallel BC$:

$$\angle BCA = \angle DAC = 60° \quad \text{[Alternate interior angles]}$$

In right-angled triangle $\triangle ABC$:

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{90}{BC}$$

$$BC = \frac{90}{\sqrt{3}} = \frac{90\sqrt{3}}{3} = 30\sqrt{3} \text{ m}$$

Substituting $\sqrt{3} = 1.73$:

$$BC = 30 \times 1.73 = 51.9 \text{ m}$$

Answer: The ship is 51.9 m away from the lighthouse.

3. Two buildings are facing each other on a road of width 12 m. From the top of the first building, which is 10 m high, the angle of elevation of the top of the second is found to be 60°. What is the height of the second building?

Solution:

Let AB be the height of the first building (10 m) and CD be the height of the second building.

Road width $BD = 12$ m.

Draw $AM \perp CD$.

In quadrilateral $AMDB$:

- $\angle B = \angle D = 90°$

- $\angle M = 90°$
- Therefore, $AMDB$ is a rectangle.

Opposite sides of rectangle are equal:

- $AB = MD = 10$ m
- $AM = BD = 12$ m

In right-angled triangle $\triangle AMC$:

$$\tan 60° = \frac{CM}{AM}$$

$$\sqrt{3} = \frac{CM}{12} \Longrightarrow CM = 12\sqrt{3} \text{ m}$$

Height of second building CD:

$$CD = CM + MD = 12\sqrt{3} + 10$$

Using $\sqrt{3} \approx 1.732$:

$$CM = 12 \times 1.732 = 20.784 \text{ m}$$

$$CD = 20.784 + 10 = 30.784 \text{ m} \quad (\approx 30.76 \text{ m})$$

Answer: The height of the second building is 30.76 m (or $10 + 12\sqrt{3}$ m).

Problem Set 6 (Page 138)

1. Choose the correct alternative answer for the following questions.

1. $\sin\theta \cdot \csc\theta = ?$

(A) 1

(Explanation: $\sin\theta \cdot \csc\theta = \sin\theta \cdot \frac{1}{\sin\theta} = 1$*)*

2. $\csc 45° = ?$

(B) $\sqrt{2}$

3. $1 + \tan^2\theta = ?$

(C) $\sec^2\theta$

4. **When we see at a higher level from the horizontal line, the angle formed is ....**

(A) angle of elevation

2. If $\sin\theta = \frac{11}{61}$, find the value of $\cos\theta$ using a trigonometric identity.

Solution:

Given: $\sin\theta = \frac{11}{61}$

By trigonometric identity:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\left(\frac{11}{61}\right)^2 + \cos^2\theta = 1$$

$$\frac{121}{3721} + \cos^2\theta = 1$$

$$\cos^2\theta = 1 - \frac{121}{3721} = \frac{3721 - 121}{3721} = \frac{3600}{3721}$$

Taking square root on both sides:

$$\cos\theta = \frac{60}{61}$$

Answer: $\cos\theta = \frac{60}{61}$.

3. If $\tan\theta = 2$, find the values of other trigonometric ratios.

Solution:

Given: $\tan\theta = 2$

1. **Find** $\sec\theta$**:**

$$1 + \tan^2\theta = \sec^2\theta \implies 1 + 2^2 = \sec^2\theta \implies \sec^2\theta = 5 \implies \sec\theta = \sqrt{5}$$

2. **Find** $\cos\theta$**:**

$$\cos\theta = \frac{1}{\sec\theta} = \frac{1}{\sqrt{5}}$$

3. **Find** $\sin\theta$**:**

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \implies \sin\theta = \tan\theta \cdot \cos\theta = 2 \cdot \frac{1}{\sqrt{5}} = \frac{2}{\sqrt{5}}$$

4. **Find** $\csc\theta$**:**

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{5}}{2}$$

5. **Find** $\cot\theta$**:**

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{2}$$

Answer:

$$\sin\theta = \frac{2}{\sqrt{5}}, \quad \cos\theta = \frac{1}{\sqrt{5}}, \quad \csc\theta = \frac{\sqrt{5}}{2}, \quad \sec\theta = \sqrt{5}, \quad \cot\theta = \frac{1}{2}$$

4. If $\boldsymbol{\sec\theta = \frac{13}{12}}$, find the values of other trigonometric ratios.

Solution:

Given: $\sec\theta = \frac{13}{12}$

1. **Find** $\cos\theta$**:**

$$\cos\theta = \frac{1}{\sec\theta} = \frac{12}{13}$$

2. **Find** $\tan\theta$**:**

$$1 + \tan^2\theta = \sec^2\theta \Longrightarrow \tan^2\theta = \left(\frac{13}{12}\right)^2 - 1 = \frac{169}{144} - 1 = \frac{25}{144} \Longrightarrow \tan\theta = \frac{5}{12}$$

3. **Find** $\cot\theta$**:**

$$\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$$

4. **Find** $\sin\theta$**:**

$$\sin\theta = \tan\theta \cdot \cos\theta = \frac{5}{12} \times \frac{12}{13} = \frac{5}{13}$$

5. **Find** $\csc\theta$**:**

$$\csc\theta = \frac{1}{\sin\theta} = \frac{13}{5}$$

Answer:

$$\sin\theta = \frac{5}{13}, \quad \cos\theta = \frac{12}{13}, \quad \tan\theta = \frac{5}{12}, \quad \csc\theta = \frac{13}{5}, \quad \cot\theta = \frac{12}{5}$$

5. Prove the following:

(1) $\boldsymbol{\sec\theta\,(1 - \sin\theta)(\sec\theta + \tan\theta) = 1}$

Proof:

$$\text{LHS} = (\sec\theta - \sec\theta\sin\theta)(\sec\theta + \tan\theta)$$

Note that $\sec\theta\sin\theta = \frac{1}{\cos\theta}\cdot\sin\theta = \tan\theta$:

$$= (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$$

$$= \sec^2\theta - \tan^2\theta = 1 = \text{RHS}$$

(Hence proved)

(2) $(\sec\boldsymbol{\theta} + \tan\boldsymbol{\theta})(1 - \sin\boldsymbol{\theta}) = \cos\boldsymbol{\theta}$

Proof:

$$\text{LHS} = \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)(1 - \sin\theta)$$

$$= \left(\frac{1 + \sin\theta}{\cos\theta}\right)(1 - \sin\theta) = \frac{1 - \sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta} = \cos\theta = \text{RHS}$$

(Hence proved)

(3) $\sec^2\boldsymbol{\theta} + \csc^2\boldsymbol{\theta} = \sec^2\boldsymbol{\theta}\cdot\csc^2\boldsymbol{\theta}$

Proof:

$$\text{LHS} = \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta\sin^2\theta}$$

$$= \frac{1}{\cos^2\theta\sin^2\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \left(\frac{1}{\cos^2\theta}\right)\left(\frac{1}{\sin^2\theta}\right) = \sec^2\theta\cdot\csc^2\theta = \text{RHS}$$

(Hence proved)

(4) $\cot^2\boldsymbol{\theta} - \tan^2\boldsymbol{\theta} = \csc^2\boldsymbol{\theta} - \sec^2\boldsymbol{\theta}$

Proof:

Using standard identities $\cot^2\theta = \csc^2\theta - 1$ and $\tan^2\theta = \sec^2\theta - 1$:

$$\text{LHS} = (\csc^2\theta - 1) - (\sec^2\theta - 1)$$

$$= \csc^2\theta - 1 - \sec^2\theta + 1$$

$$= \csc^2\theta - \sec^2\theta = \text{RHS}$$

(Hence proved)

(5) $\boldsymbol{\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta}$

Proof:

$$\text{LHS} = \tan^2\theta\,(\tan^2\theta + 1)$$

$$= \tan^2\theta \cdot \sec^2\theta \quad [\because 1 + \tan^2\theta = \sec^2\theta]$$

$$= (\sec^2\theta - 1) \cdot \sec^2\theta \quad [\because \tan^2\theta = \sec^2\theta - 1]$$

$$= \sec^4\theta - \sec^2\theta = \text{RHS}$$

(Hence proved)

(6) $\boldsymbol{\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta}$

Proof:

$$\text{LHS} = \frac{(1+\sin\theta) + (1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{2}{1-\sin^2\theta}$$

$$= \frac{2}{\cos^2\theta} \quad [\because 1 - \sin^2\theta = \cos^2\theta]$$

$$= 2\sec^2\theta = \text{RHS}$$

(Hence proved)